Exploring Different Numerical Bases

The numeral system we use is base ten. We use ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and the value of a digit depends on its position in a number. For example,

$$2016 = 2 \cdot 10^3 + 0 \cdot 10^2 + 1 \cdot 10^1 + 6 \cdot 10^0$$

Numbers can be written in any number base $n \geq 2$.

Base n uses digits 0, 1, 2, ..., $n - 1$.

Decimal (Base ten)

10^6	10^5	10^4	10^3	10^2	10^1	10^0	Base with power
1,000,000	100,000	10,000	1,000	100	10	1	Place value
							Digits used 0-9

Binary (Base two)

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	Base with power
128	64	32	16	8	4	2	1	Place value
								Digits used 0-1

Ternary (Base three)

3^5	3^4	3^3	3^2	3^1	3^0	Base with power
243	81	27	9	3	1	Place value
						Digits used 0-2

To convert to another base from base ten:

Find the highest power of the base that fits into the given number (can fit more than once!)
- Subtract this power from the number as many times as it fits.
- Repeat with the new number.

Example: Convert 124_{10} to base three.

81 fits in once 124-81 = 43

27 fits in once 43-27 = 16

9 fits in once 16-9 = 7

3 fits in twice 7-2(3) = 1

1 fits in once 1-1 = 0 Therefore, $11121_3 = 124$ in our numeral system